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Comments on the 2011 Shaw Prize in Mathematical Sciences -- An Analysis of Collectively Formed Errors in Physics

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GJSFR-A Classification : 04.20.-q, 04.20.Cv

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Strictly as per the compliance and regulations of :

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Keywords : Nobel Prize; general relativity; Einstein equation, Riemannian Space; the non-existence of dynamic solution; Galileo.

> "Science sets itself apart from other paths to truth by recognizing that even its greatest practitioners sometimes err. … We recognize that our most important scientific forerunners were not prophets whose writings must be studied as infallible guides—they were simply great men and women who prepared the ground for the better understandings we have now achieved." -- S. Weinberg, Physics Today, November 2005.

I. Introduction

The Shaw Prize, named after Hong Kong film and television magnate Run Run Shaw, each year recognizes innovation in three fields—astronomy, medicine and mathematics—with three awards of US$1 million each. It's often called Asia's Nobel Prize, though it's a global honor; this year's winners, announced by the Shaw Prize Foundation in Hong Kong, are all from Europe and the U.S.

However, as a new prize since 2002, the committee also makes some errors in choosing the winners and recognizing their merits. This year of 2011, a half of the prize in mathematics is awarded to Richard S. Hamilton, a distinguish mathematician for his work on the Ricci flow that lays down the foundation to prove the Poincare conjecture. Unfortunately the Shaw Prize also made a mistake by awarding the other half prize to Christodoulou for his work, based on obscure errors, against the honorable Gullstrand [1, 2] of the 1921 Nobel Committee.1) Although Christodoulou has misled many including the 1993 Nobel Committee [3], his errors are now well-established and they have been illustrated with mathematics at the undergraduate level [4, 5]. Thus, it is possible to neutralize this disservice to science.

The official announcement for awarding them is,2) "for their highly innovative works on nonlinear partial differential equations in Lorentzian and Riemannian geometry and their applications to general relativity and topology." Christodoulou claimed in his Autobiography that his work is essentially based on two sources: 1) The claims of Christodoulou and Klainerman on general relativity as shown in their book ***The Global Nonlinear Stability of the Minkowski Space*** [6]; 2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete [7]. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman [6]. However, such a relation was not clear until 1995 when this implicit assumption was proved incorrect [8].

Due to inadequate mathematical background in comparison with Gullstrand, physicists including Einstein [9], Pauli [10], Misner, Thorne & Wheeler [11], etc. believed that, as in the case of linear equation, the nonlinear Einstein equation should have the bounded dynamic solutions. This view seems to be supported by solutions of the static case, and also a linearization of the Einstein equation. Thus, Gullstrand's suspicion on validity of Einstein's calculation was not generally accepted. Although nobody can provide valid evidence to support Einstein's view, some went so far as to claim that Gullstrand had the advantage because he was Swedish.

The fact is, however, that Einstein's equation cannot have a bounded dynamic solution [8]. Also, the singularity theorems of Penrose and Hawking [7] are actually irrelevant to physics because they use an

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unphysical implicit assumption [8] that violates the principle of causality [12].

Historically, in 1921 Gullstrand [1, 2] conjectured and sustained that Einstein's equation may not have a dynamic solution. In 1993 Christodoulou and Klanerman [6] claimed that bounded dynamic solutions were constructed. However, in 1995, as a continuation of the non-existence of plane-wave solutions, it is proven that there are no dynamic solutions or wave solutions for Einstein's equation [8]. Moreover, for the dynamic case, linearization to obtain an approximation is not valid in mathematics. Thus, Gullstrand's conjecture is proven to be correct. Subsequently their book [6] was severely criticized [13-15] while it is still classified as No. 41 in the Princeton Mathematical Series. Moreover, these criticisms are also supported by the fact that there is no bounded dynamic solution in the literature. The physical reason is identified as that, for a dynamic case, Einstein's equation violates the principle of causality.[3)]

Nevertheless, Nobel Laureate't Hooft attempted to challenge Gullstrand with a bounded time-dependent solution in 2004, but was defeated because his solution also violates the principle of causality [4, 16].[4)] In addition, Wald [7] believed that perturbation approach was always valid to obtain an approximate solution. Meanwhile Professor P. Morrison of MIT met Nobel Laureate Professor J. Taylor of Princeton University several times to discuss problems on the dynamic solution [17], but Taylor failed to defend their calculation of the binary pulsars [18, 19].[5)]

To facilitate theorists, whose views are based on earlier mathematical errors, in understanding the absence of dynamic solutions and wave solutions of the Einstein equation, a review paper on this subject with counter examples being understandable at the undergraduate level [4, 5] was published in 2011. Thus, the errors of Christodoulou on general relativity are further clarified and no longer in doubt.[6)] *A basic rule in mathematics learned in my undergraduate years is that one must be able to support his mathematical statements with examples. It seems that some theorists including members of the Selection Committee of the Shaw Prize have forgotten this simple rule.*

Both Christodoulou and Hamilton have cited the influence of Fields Medalist (1982) S. T. Yau. However, there are some important details that the Shaw Committee failed to notice. While supporting Hamilton in the recent participation in solving the Poincare conjecture, Yau has in effect withdrawn his support to Christodoulou by declaring his loss of earlier interest on the related work [6] as acknowledged. Nevertheless, Yau may still not understand that general relativity was not yet self-consistent [19] since he has not made any modification on the positive mass theorem of Schoen and Yau [20, 21] that also used the invalid implicit assumption of unique sign for all coupling constants, as Penrose and Hawking did [7].[4)] Since acceptance of invalid claims and misinterpretations has reached the level of Fields Medalists [22],[7)] the mistake of the awarding a prize to a mathematician for his errors should no longer be a great surprise!

Having been educated in Hong Kong, I feel the need to point out this error of the Shaw Committee that is clearly against the wish of Mr. Shaw, to award advancements in sciences. To help the scientific community overcome these errors, which have involved the 1993 Nobel Committee, Caltech, Harvard University, Princeton University, the Physical Review, and the Royal Society, etc., it would be necessary to point out the literature related to the errors in mathematics and physics. Moreover, Christodoulou should be informed formally in an open letter that his work is still incorrect.

II. Open Letter to Christodoulou

The errors of Christodoulou are described in an open letter to him as follows:

Prof. Demetrios Christodoulou,Professor of Mathematics and Physics
HG G 48.2, ETH-Zentrum
CH-8092 Zürich
Switzerland
E-Mail: demetri@math.ethz.ch

Dear Professor Christodoulou:

Congratulations for the Shaw Prize in mathematics! It is an honor that you are able to share a prize with a distinguished mathematician Richard Streit Hamilton, a professor at Columbia University.

I have been looking for you since 2000 after I have read your book [6] coauthored with Klainerman. I find that your proof on the existence of a dynamic solution is incomplete because you failed to show that the set of your initial conditions is non-empty. In fact, other reviewers say the same thing indirectly that your first chapter is not comprehensible. I have asked your coauthor Klainerman to provide the missing information, but he declined. I was also informed that you were no longer at Princeton University, and have returned to Greece.

Nevertheless, if your errors were unclear because of your complicated calculations, your errors can now be understood much easier because they can be illustrated with mathematics at the undergraduate level. I would like to inform you that a Nobel Laureate't Hooft had attempted to defend your work, but failed since he does not understand the related physics [4, 15].[4)] In fact, your errors are also well known by now because I have written a paper, "Linearization of the Einstein Equation and the 1993 Press Release of the Nobel Prize in Physics" [5]. This paper shows how the errors in your erroneous book [6] are criticized by other scientists and how your errors have misled others as shown in the errors of the 1993 press release of the Nobel Committee [3].

I have reported my paper in the 18th Annual Natural Philosophy Alliance Conference, July 6-9, 2011, at the

University of Maryland, College Park, USA; and my paper is well received because of its clarity that also explains your mathematical errors well. I can say this with such a definitive tone because there are explicit examples that confirm your errors. For your perusal, a copy of the file of this paper is attached.

The basic problem in terms of physics is that just as in Maxwell's classical electromagnetism [23], there is also no radiation reaction force in general relativity. Although an accelerated massive particle would create radiation [24], the metric elements in the geodesic equation are created by particles other than the test particle [9].[8)]

This problem is manifested by the fact that there is no dynamic solution for the Einstein equation [8, 12, 13, 18], which does not include the gravitational energy-stress tensor of its gravitational waves in the sources.[9)] Thus, to fit the data,[10)] it is necessary to modify the Einstein equation [8] to

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K\left[T(m)_{\mu\nu} - t(g)_{\mu\nu}\right] \qquad (1)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity. For radiation, the tensor $t(g)_{\mu\nu}$ is equivalent to Einstein's notion of gravitational energy-stress. Because a wave carries energy and momentum in vacuum, it is necessary to have such an additional tensor term. However, Einstein's notion is a pseudo-tensor and can become zero by choosing a suitable coordinate system, but the energy-momentum of a radiation cannot be zero, and thus must be a tensor [8].

In conclusion, the Einstein equation cannot have a dynamic solution because the principle of causality is violated! Thus, your work on general relativity is clearly incorrect in terms of both mathematics and physics. Therefore, please rectify these errors to overcome the rejection of the brilliant work of Gullstrand [1, 2]. You owe the scientific world for rectifying these errors. Moreover, your errors are the main obstacles to theoretical progress in general relativity that have been experimentally confirmed [19]!

The Wheeler School needs to rectify their errors, but they have neither the background in mathematics nor the will to rectify their mistakes [22]. In addition, it would be to your benefit to rectify these errors. You are young and thus still have a chance to take a more honorary role in science! Besides, there is still no authority in general relativity yet [19].

I would suggest that you use your share of the award money to help assemble a team to develop general relativity and to rectify the remaining errors. This would be a very fruitful field since a new force has been discovered [19].

Best wishes!

Sincerely yours,

C. Y. Lo

In the next section, some details of Christodoulou's mathematical errors in logic are provided.

III. Some Remarks on the Errors of Christodoulou in Mathematics and Physics

The book of Christodoulou & Klanerman [6] is confusing (see Appendix A). Their main Theorem 1.0.3 states that any strongly asymptotically flat (S.A.F.) initial data set that satisfies the global smallness assumption leads to a unique globally hyperbolic asymptotically flat development. However, because the global smallness assumption has no dynamic requirements in their proofs, there is no assurance for the existence of a dynamic S.A.F. initial data set [13]. Thus, the existence of a bounded dynamic initial set is assumed only, and their proof is at least incomplete.

Perlik [14] complained, "What makes the proof involved and difficult to follow is that the authors introduce many special mathematical constructions, involving long calculations, without giving a clear idea of how these building-blocks will go together to eventually prove the theorem. The introduction, almost 30 pages long, is of little help in this respect. Whereas giving a good idea of the problems to be faced and of the basic tools necessary to overcome each problem, the introduction sheds no light on the line of thought along which the proof will proceed for mathematical details without seeing the thread of the story. This is exactly what happened to the reviewer." Thus, their claim on "dynamic" solutions was met with wide spread skeptics [14]. They assume the existence of a bounded initial set to prove the existence of a bounded solution. *Moreover, his initial condition has not been proven as compatible with the Maxwell-Newton approximation which is known to be valid for weak gravity* [13].

The above claim is similar to what Misner et al. [11] did. They claimed their plane-wave equation,

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \qquad (2)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = t - x$ has a bounded plane-wave solution as follows:

$$ds^2 = dt'^2 - dx'^2 - L^2(e^{2\beta}dy^2 + e^{-2\beta}dz^2). \qquad (3)$$

Careful calculation with undergraduate mathematics shows that this is impossible [4, 5]. Thus, many others like Chistodoulou made or accepted an invalid claim, but was unaware of errors at the undergraduate level.

An example to illustrate a violation of the principle of causality is the solution of Einstein's cylindrical symmetric wave solution [16]. The metric of Bondi, Pirani & Robinson [25] also violates this principle, and is as follows:

$$ds^2 = \exp(2\phi)(d\tau^2 - d\xi^2) - u^2[\mathrm{ch}2\beta\,(d\eta^2 + d\zeta^2) + \mathrm{sh}2\beta\cos 2\theta\,(d\eta^2 - d\zeta^2) - 2\mathrm{sh}2\beta\sin 2\theta\, d\eta d\zeta], \qquad (4)$$

where ϕ, β, θ are functions of u ($= \tau - \xi$). It satisfies the differential equation (i.e., their Eq. [2.8]),

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$$2\phi' = u(\beta'^2 + \theta'^2 sh^2 2\beta). \qquad (5)$$

This metric is unbounded. When the time-dependent factors are reduced to constant (i.e.,$\phi' = \beta' = \theta' = 0$), this metric cannot be reduced to the flat metric as the case of Einstein's "wave" [16]. Thus, the Royal Society like Christodoulou, also claimed dynamic solutions, but was unaware of a violation of the principle of causality in physics.

Another "plane wave", which is intrinsically unphysical, is the metric accepted by Penrose [26] as follows:

$$ds^2 = du\, dv + Hdu^2 - dx_i\, dx_i, \quad \text{where} \quad H = h_{ij}(u)\, x_i\, x_j \quad (6)$$

where $u = ct - z$, $v = ct + z$, $x = x_1$ and $y = x_2$, $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$. This metric satisfies the harmonic gauge. The cause of metric (6) can be an electromagnetic plane wave. Metric (6) satisfies

$$\eta^{\alpha\beta}\, \partial_\alpha \partial_\beta\, \gamma_{tt} = -2\{h_{xx}(u) + h_{yy}(u)\} \quad \text{where} \quad \gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \qquad (7)$$

However, this does not mean that causality is satisfied although metric (14) is related to a dynamic source. The violation of the principle of causality of this metric is due to containing unphysical parameters [16].

Many theorists assume a physical requirement would be unconditionally satisfied by the Einstein equation [19]. Apparently, Christodoulou adapted such a view. As shown, his mathematical analysis is also not reliable at the undergraduate level although Christodoulou claimed to have such a strong interest in his autobiography. In addition, Christodoulou does not understand the difference between mathematics and physics. According to the principle of causality in physics, a bounded dynamic solution should exist, but this does not necessarily mean mathematically that the Einstein equation has such a solution.

Gullstrand was not the only theorist who questioned the existence of the bounded dynamic solution for the Einstein equation. As shown by Fock [27], any attempt to extend the Maxwell-Newton approximation (the same as the linearized equation with mass sources [8]) to higher approximations leads to divergent terms. In 1993, it has been proven [8, 28] that for a dynamic case the linearized equation as a first order approximation, is incompatible with the nonlinear Einstein field equation. Moreover, the Einstein equation does not have a dynamic solution for weak gravity unless the gravitational energy tensor with an anti-gravity coupling is added to the source (see also eq. [1]). The necessity of an anti-gravity coupling term manifests why a bounded wave solution is impossible for Einstein's equation.

After it has been shown that there is no bounded dynamic solution for the Einstein equation [8], in 1996 Perlick published a book review in ZFM, pointing out that Christodoulou and Klanerman have made some unexpected mistakes, and their mathematical proof is difficult to follow, and suggested their main conclusion may be unreliable. However, to many readers, a suggestion of going through more than 500 pages of mathematics is not a very practical proposal.

Their book [6] was accepted because it supports and is consistent with existing errors as follows:

1) It supports errors that created a faith on the existence of dynamic solutions of physicists including Einstein etc.
2) Due to the inadequacy of the mathematics used, the book was cited before 1996 without referring to the details.
3) Nobody suspected that professors in mathematics and/or physics could made mistakes at the undergraduate level.
4) Because physical requirements were not understood, unphysical solutions were accepted as valid [26, 29-31]. Thus, in the field of general relativity, strangely there is no expert almost 100 years after its creation.

In physics, a dynamic solution must be related to dynamic sources, but a "time-dependent" solution may not necessarily be a physical solution [4, 16, 25].[4)] To begin with, their solutions are based on dubious physical validity [13]. For instance, their "initial data sets" can be incompatible with the field equation for weak gravity. Second, the only known cases are static solutions. Third, they have not been able to relate any of their constructed solutions to a dynamic source. In pure mathematics, if no example can be given, such abstract mathematics is likely wrong [32].

In fact, there is no time-dependent example to illustrate the claimed dynamics (see Appendix B and [13]). In 1953 Hogarth [33] already conjectured that a dynamic solution for the Einstein equation does not exist. Moreover, in 1995 it is proven impossible to have a bounded dynamic solution because the principle of causality is violated [8].

IV. The Shaw Prize and Her Governing Committees

The Shaw Prize is governed by a Board of Adjudicaters, under which there are three selection committees of astronomy, medicine and mathematics. Each committee selects the winners for each prize.

Board of Adjudicators

Chairman: Professor Chen-Ning Yang <cnyang@tsinghua.edu.cn>

Vice Chairman: Professor Kenneth Young <kyoung@cuhk.edu.hk>

Members: Professor Jiansheng Chen hlhl@public.sti.ac.cn, Professor Yuet-Wai KAN <iomwww@nas.edu>, and Professor Peter C. Sarnak <sarnak@math.ias.edu>.

Selection Committee for the Shaw Prize in Mathematical Sciences

Chairman: **Professor Peter C. Sarnak**
sarnak@math.ias.edu
Professor of Mathematics
Princeton University and Institute for Advanced Study
USA
Members: **Sir Michael Atiyah** M.Atiyah@ed.ac.uk
Honorary Professor
School of Mathematics
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Professor David Kazhdan
kazhdan@math.huji.ac.il
The Einstein Institute of Mathematics
Faculty of Science
The Hebrew University of Jerusalem
Israel
Professor Yum-Tong Siu
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Silver Professor of Computer Science and Mathematics
Courant Institute of Mathematical Sciences
New York University
USA

The selection of mathematicians for the prize lies in this selection committee. However, if you check the background, it seems none of the members has an adequate research background in general relativity. The award speech for mathematics (see Appendix C) was made by Margaret H. Wright, who is in Computer Science but not functional analysis. From her speech, it is clear that the works of Christodoulou and Hamilton are actually not related. Thus, one may wonder who initiated the nomination of Christodoulou.

It is known that, based on gauge invariance, Professor C. N. Yang is against the view of Zhou Pei-Yuan [34, 35] on invalidity of Einstein's covariance principle. However, according to S. Weinberg on gauge theories [36] and direct research in general relativity [19],[11)] Yang is proven wrong; but the work of Christodoulou is in another area.

It seems that, inheriting from Christodoulou, Penrose, and 't Hooft, etc. a failure in distinguishing mathematics and physics,[9),10)] the Selection Committee in Mathematical Sciences leads to giving an award for mathematical errors. They seem to neglect whether there are supporting examples with valid dynamic sources;[4)] and also do not understand the related physical requirements. Their misjudgment should have been expected since they seem to be unaware of the known errors of Hawking and Penrose in physics [8] as at least a dozen of Nobel Laureates had made such errors.

V. Discussions and Conclusions

However, although the Shaw Prize is directly responsible for this error, there are theorists, starting from Einstein and Hilbert in 1915 [37], helping its unusual long gestation of more than 95 years because of inadequate knowledge in the non-linear equation [19]. It took a genius such as Gullstrand[1)] to discover this error, but it was still not believed among many theorists due to their inadequacy in mathematics. Meanwhile, this error was made obscure by other errors such as the failure in distinguishing the difference between mathematics and physics [19]. Such a failure is responsible for rejecting invalidity of Einstein's covariance principle, a discovery of Zhou Pei-Yuan [34, 35]. This confusion also leads to an inadequate understanding on the physical principles,[3)] and this problem leads to further errors in general relativity [19]. We should learn from errors of Penrose etc. [7, 26] to prevent further errors in the future.

This analysis shows that the misunderstanding of physics on the existence of dynamic solutions is the root of other related errors. Because of background in mathematics, and/or a failure to distinguish the difference between mathematics and physics and etc., only some theorists are able to see the errors of Christodoulou [8, 14, 15, 17]. Instead, many are misled by the invalid claims of Christodoulou, and failed to see counter examples [4, 5].

For instance, his errors are related to the implicit assumption of unique sign for all coupling constants [8], which is used in the singularity theorems of Hawking and Penrose [7] that lead to the speculation of an expanding universe [38]. The errors of Christodoulou also supports the invalid speculation that $E = mc^2$ is unconditionally valid [39, 40]. In turn, this supports that gravity is always attractive, a foundation of the theory of black holes [41]. Such errors lead also to the acceptance of unbounded solution, implicitly rejecting the principle of causality [16], etc. Now, the errors of Christodoulou lead to an award of the Shaw Prize in mathematics for standing in the way of theoretical progress.[12)]

It is expected that this paper would recover the honor of Gullstrand. Because of accumulated mistakes by the institutes, a highly competent theorist[1)] could be defeated by an incompetent.[13)] In a way, this is an inevitable result of long time accumulation of errors. However, the Shaw Prize Committee has the responsibility for exposing these errors although she is not solely responsible for their creation. It seems that frontier physicists should pay more attention to physical principles and have a better education in pure mathematics. Moreover, in view of the errors once prevailing in general relativity,[14)] the communication between mathematicians and physicists should be further strengthened.

VI. Acknowledgments

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Appendix A : A Book Review on "The Global Nonlinear Stability of the Minkowski Space".

This book review originally appeared in ZfM [14] in 1996; and, with the kind permission of its Editor, B. Uegner, will be republished in the journal, GRG [15] again with the editorial note, "One may extract two messages: On the one hand, (by seeing e.g. how often this book has been cited), the result is in fact interesting even today, and on the other hand: There exists, up to now no generally understandable proof of it." For the convenience of the reader, this review is provided as an appendix. The review is as follows:

"For Einstein's vacuum field equation, it is a difficult task to investigate the existence of solutions with prescribed global properties. A very interesting result on that score is the topic of the book under review. The authors prove the existence of globally hyperbolic, geodesically complete, and asymptotically flat solutions that are close to (but different from) Minkowski space. These solutions are constructed by solving the initial value problem associated with Einstein's vacuum field equation. More precisely, the main theorem of the book says that any initial data, given on R^3, that is asymptotically flat and sufficiently close to the data for Minkowski space give rise to a solution with the desired properties. In physical terms, these solutions can be interpreted as space-times filled with source-free gravitational radiation. Geodesic completeness means that there are no singularities. At first sight, this theorem might appear intuitively obvious and the enormous amount of work necessary for the proof might come as a surprise. The following two facts, however, should caution everyone against such an attitude. First, it is known that there are nonlinear hyperbolic partial differential equations (e.g., the equation of motion for waves in non-linear elastic media) for which even arbitrarily small localized initial data lead to singularities. Second, all earlier attempts to find geodesically complete and asymptotically flat solutions of Einstein's vacuum equation other than Minkowski space had failed. In the class of spherically symmetric space-time and in the class of static space-times the existence of such solutions is even excluded by classical theorems. These facts indicate that the theorem is, indeed, highly non-trivial. Yet even in the light of these facts it is still amazing that the proof of the theorem fills a book of about 500 pages. To a large part, the methods needed for the proof are rather elementary; abstract methods from functional analysis are used only in so far as a lot of L^2 norms have to be estimated. What makes the proof involved and difficult to follow is that the authors introduce many special mathematical constructions, involving long calculations, without giving a clear idea of how these building-blocks will go together to eventually prove the theorem. The introduction, almost 30 pages long, is of little help in this respect. Whereas giving a good idea of the problems to be faced and of the basic tools necessary to overcome each problem, the introduction sheds no light on the line of thought along which the proof will proceed for mathematical details without seeing the thread of the story. This is exactly what happened to the reviewer."

"To give at least a vague idea of how the desired solutions of Einstein's vacuum equation are constructed, let us mention that each solution comes with the following: (a) a maximal space-like foliation generalizing the standard foliation into surfaces t = const. in Minkowski space; (b) a so-called optical function u, i.e. a solution u of the eikonal equation that generalizes the outgoing null function u = r - t on Minkowski space; (c) a family of "almost conformal killing vector fields on Minkowski space. The construction of these objects and the study of their properties require a lot of technicalities. Another important tool is the study of "Bianchi equations" for "Weyl tensor fields". By definition, a Weyl tensor field is a fourth rank tensor field that satisfies the algebraic identities of the conformal curvature tensor, and Bianchi equations are generalizations of the differential Bianchi identities."

"In addition to the difficulties that are in the nature of the matter the reader has to struggle with a lot of unnecessary problems caused by inaccurate formulations and misprints. E.g., "Theorem 1.0.2" is not a theorem but rather an inaccurately phrased definition. The principle of conservation of signature" presented on p. 148 looks like a mathematical theorem that should be proved; instead, it is advertised as an "heuristic principle which is essentially self-evident." For all these reasons, reading this book is not exactly great fun. Probably only very few readers are willing to struggle through these 500 pages to verify the proof of just one single theorem, however interesting."

"Before this book appeared in 1993 its content was already circulating in the relativity community in form of a preprint that gained some notoriety for being extremely voluminous and extremely hard to read. Unfortunately, any hope that the final version would be easier to digest is now disappointed. Nonetheless, it is to be emphasized that the result presented in this book is very important. Therefore, anyone interested in relativity and/or in nonlinear partial differential equations is recommended to read at least the introduction."

Note that the above review actually suggests that problems would be adequately identified in the introduction. As shown in the present paper, the possible nonexistence of their dynamic solutions and its incompatibility with Einstein's radiation formula can be discovered in their introduction. Their book has often been cited [42-52], in spite of the invalid "proof". Note, however, such citations in some journals have stopped since 1996.

From this review, what the Shaw Prize claimed as "for their highly innovative works on nonlinear partial differential equations in Lorentzian and Riemannian geometry and their applications to general relativity and topology.", in the case of Christodoulou, seems to be just a euphemism for a highly confusing and incomprehensible presentation.

Appendix B : The Smallness Assumption and the S.A.F Initial Data Condition

In this Appendix B, it is pointed out that a dynamic strongly asymptotically flat (S.A.F.) condition need not necessarily exist. Also, it is strange that the "physical" solutions are constructed with only mathematical considerations.

In their book, without physical considerations, Christodoulou and Klainerman wrote:

"Our construction requires initial data sets that satisfy, in addition to the constraint equations, the maximal condition tr k = 0 (1.0.10). We will refer to them as maximal in what follows:"

> "To make the statement of our main theorem precise, we need also to define what we mean by the global smallness assumption. Before stating this condition, we assume the metric g to be complete and we introduce the following quantity:

$$Q(x_{(0)}, b) = \sup_{\Sigma} \{b^{-2}(d_0^2 + b^2)^3 |Ric|^2\}$$

$$+ b^{-3}\{ \int_{\Sigma} \sum_{l=0}^{3} (d_0^2 + b^2)^{l+1} |\nabla^l k|^2 + \int_{\Sigma} \sum_{l=0}^{3} (d_0^2 + b^2)^{l+3} |\nabla^l B|^2\}$$

where $d_0(x) = d(x_{(0)},x)$ is the Riemannian geodesic distance between the point x and a given point $x(0)$ on Σ, b is a positive constant, $|Ric|^2 = R^{ij}R_{ij}$, ∇^l denotes the 1-covariant derivatives, and B is the symmetric, traceless 2-tensor tensor.

The symmetry and traceless of B follow immediately from the twice-contracted Bianchi identities $\nabla^i R_{ij} - ½ \nabla_i R = 0$. In the fact we can write $B_{ij}= (1/2)(\in_i^{ab}\nabla a \hat{R}_{jb} + \in_j^{ab}\nabla_a \hat{R}_{ib})$, where $\hat{R}_{ij}$ is the traceless part of R_{ij}, $R_{ij}= \hat{R}_{ij}+1/3R g_{ij}$.

Theorem 1.0.2 : (The Global Smallness Assumption) We say that a strongly asymptotically flat (S.A.F) initial data set, (Σ, g, k), satisfies the global smallness assumption if the metric g is complete and there exists a sufficiently small positive ∈ such that

$$\inf_{x_{(0)}\in\Sigma, b\geq 0} Q(x_{(0)}, b) < \in \qquad (1.0.15)$$

Theorem 1.0.3 (Second Version of the Main Theorem) Any strongly asymptotically flat, Maximal, initial data set that satisfies the global smallness assumption 1.0.15 leads to a unique, globally hyperbolic, smooth, and geodesically complete solution of the Einstein-Vacuum equation foliated by a normal maximal time foliation. Moreover, this development is globally asymptotically flat.

Remark 1.1 : In view of the scale invariance property of the Einstein-Vacuum equations, any initial data set Σ, g, k for which $Q(x_0, b) < \in$ can be rescaled to the new initial data set ä, g', k' with $g' = b^{-2}g$, $k' = b^{-1}k$ for which $Q(x_0, 1) < \in$. The global existence for the new set is equivalent to the global existence for the original set. This is due to the fact that the developments g, g' of the two sets are related by $g' = b^{-2}g$. It thus suffices to prove the theorem under the global smallness assumption

$$\inf_{x_{(0)}\in\Sigma} Q(x_{(0)}, b) < \in."$$

then, they prove that for given arbitrary solutions $\tilde{g}$, $\tilde{k}$ to the equations

$$tr_{\tilde{g}}\, \tilde{k} = 0, \qquad (1.0.16a)$$

$$\tilde{\nabla}^j \tilde{k}_{ji} = 0 \qquad (1.0.16b)$$

which are invariant with respect to the conformal transformation, this suffices to insure an initial data set (Σ, g, k) satisfying the S.A.F. condition if

$$\tilde{g}_{ij} = \delta_{ij} + o_4(r^{-3/2}), \text{ and } \quad \tilde{k}_{ij} = o_3(r^{-5/2})$$

and the negative part of $\hat{R}$ satisfies the smallness condition.

Moreover, g and k satisfy the global smallness assumption of the theorem provided that the metric $\tilde{g}$ is complete and that there exists a mall positive ∈ such that

$$\inf_{x(0)\in\Sigma, a\geq 0} \{ \sup_{\Sigma} (\bar{d}_0^{\,2} + a^2)^3 / \tilde{Ric} /^2\}$$

$$+ \int_{\Sigma} \sum_{l=0}^{3} (\bar{d}_0^{\,2} + a^2)^{l+2} | \tilde{\nabla}^{\ell} \tilde{Ric} |^2 + \int_{\Sigma} \sum_{l=0}^{3} (\bar{d}_0^{\,2} + a^2)^{l+1} | \tilde{\nabla}^{\ell} \tilde{k} |^2 < \in$$

where $\bar{d}_0(x)$ $(= \bar{d}_0)$ denotes the Riemannian geodesic distance relative to $\tilde{g}$ between the point x and a given point x(0) on Σ. Thus, it remains to discuss whether the equation 1.0.16a and 1.0.16b have solutions.

However, because condition (1.0.15), (1.0.10) and equation (1.0.16) have no dynamic requirements in their proofs, there is no assurance for the existence of a dynamic S.A.F. initial data set. If such a dynamic set does not exist, then the entire book is just for the static case! *Moreover, when a solution is assumed to be* *bounded, it would be automatically reduced to the static case.* Another basic problem of Christodoulou is that his understanding in physics is also fundamentally inadequate. In their book [6] of 500 pages, they did not address the sources of constructed solutions. If this is not due to their careless oversight, they may have failed to relate their solution to dynamic sources.

Appendix C : Speech by Professor Margaret H Wright (Member of Mathematical Sciences Selection Committee)

The Speech by Professor Margaret H Wright manifests that many failed to understand the non-existence of dynamic solution for Einstein's equation.[14] Her speech before awarding the Shaw Prize for mathematics is as follows:

As in recent years, many outstanding and worthy nominations were made this year for the Shaw Prize in Mathematical Sciences. However, two names – Demetrios Christodoulou and Richard Hamilton – quickly rose to the top. The primary works of both involve the global behavior of nonlinear evolution equations in geometry, a large and active area in modern mathematics and mathematical physics. The central theme of their work is the formation of singularities for geometric evolution equations, a crucial question in general relativity or Riemannian geometry.

Demetrios's contributions are in mathematical physics – in particular, partial differential equations describing physical phenomena. His study of the behavior of solutions to Einstein's equations in general relativity has shaped our understanding of the formation of singularities such as black holes, as well as basic issues such as the stability of the Minkowski-space time. He is unique in having a deep understanding and intuition about the underlying physics while at the same time being a brilliant (mathematical) analyst. This combination of traits has led him to rigorous treatments and discoveries of unexpected phenomena. Along the way he has solved problems that had resisted progress for many years.

Richard has made many contributions to geometric analysis. In particular, his Ricci Flow, introduced to describe low-dimensional positively curved spaces, is one of the great gifts to modern mathematics. Over the past three decades Richard has led the way by developing a host of techniques to study the long-time behavior of his Ricci flow and to deal with singularities. His ideas have led to many results in geometry, topology, and the physics connected with curvature flows. The most spectacular of these is Grigori Perelman's proof of Thurston's Geometrization Conjectures (including Poincare' as a special case),[15] which builds on Richard's theory of Ricci flow. The resulting classification of three-dimensional shapes constitutes one of the finest achievements in mathematics.

To sum up, the profound and innovative works of Demetrios and Richard are very hard-earned, achieved only by sticking to their ideas and beliefs over a long period of time. Their efforts are an inspiration to us all.

Comments from the author :

From this speech, while the evaluation of Richard Hamilton is valid, the Selection Committee does not understand the mathematics of Christodoulou and related issues in physics. Perhaps, the selection for Christodolou may be a little too quick. In fact, his views were shared by others, and were severely criticized [13-15]. As shown in their book [6], he studied solutions of a field without addressing the related sources [6]. Christodoulou should have known that, a time-dependent solution has no meaning unless it is related to dynamic sources. He failed to tell the difference between mathematics and physics, and to justify his assumption in physics [14]. It is clear that he does not understand physics and the principle of causality. Moreover, he made crucial mathematical errors at the undergraduate level [6, 11].

His errors in mathematics prevent others from seeing that the implicit assumption of Penrose and Hawking on the unique sign of all coupling constants is invalid [8]. This invalid physical assumption is crucial to their singularity theorems that led others to support the notion of black holes and to claim incorrectly that general relativity is invalid for microscopic phenomena [7, 19]. Thus, the claims of Christodoulou are major obstacles to the progress in physics.

Christodoulou should have given explicit solutions, *instead of just making an invalid claim as Misner et al did* [11]. He should have shown that their solutions were compatible with physically valid sources; *and this was what't Hooft failed* [4]. Moreover, he should have checked whether their solutions satisfy all physical requirements; and this was also what Bondi et al. [25], Penrose [26], 't Hooft as well as the Physical Review [16] and the Proceedings of the Royal Society A etc. have failed. The Shaw Prize Selection Committee also failed to see these problems because they do not understand physics. Thus, just like many others, the Prize Committee seems to blindly follow mistakes in the publications of Princeton and Einstein such as references [6], [11], and [53] etc. without the necessary deliberations. Had members of the Selection Committee tried to find an example of the dynamic solution that could support the claims of Christodoulou, they would have found his errors. Their award to an erroneous work is clearly a disservice to science.

In short, D. Christodoulou is incompetent in both mathematics and physics. Nevertheless, a combination of such traits in his special way together with prevailing misconceptions has led to crucial errors that were accepted by many theorists because of their own bias in physics and/or inadequacy in mathematics. Prof. Wright, as the speaker for a prize in analysis, is in Computer Science. It seems, the work of the honorable Gullstrand [14, 15] has been ignored, and the committee was also unaware of the recent important theoretical and experimental developments [21, 54].

Endnotes

1. A. Gullstrand won a Nobel Prize in 1911, was a member of the Nobel Physics Committee of the Swedish Academy of Sciences in 1921, and was the Chairman of the committee (1922-1929). Because of his work [1, 2], Einstein's Nobel Prize was for his discovery on the law of the photoelectic effect, but not general relativity. However, the confirmation of Gullstrand's ingenuity is a natural result of long-time hard work [8] from another area.
2. All information on Shaw Prizes can be found from their announcements in Google.
3. In disagreement with Einstein & Rosen, the Physical Review accepted "wave" solutions with unbounded amplitude as valid in physics because of being unaware of the violation of the principle of causality [55].
4. Although the time-dependent solution of't Hooft is bounded, it violates the principle of causality since his "solution" has no valid sources [4]. He failed in distinguishing a difference between mathematics and physics [4, 56]. In his 1999 Nobel Speech [57],'t Hooft showed misunderstandings of the notion of mass and special relativity.
5. Their calculation of the gravitational waves of binary pulsars failed because Einstein's equation does not have a bounded dynamic solution [8], which is necessary for their calculations of the gravitational radiation.
6. Many errors are actually created by the so-called "experts" [19]. For instance, the notion of local Lorentz invariance comes from the misinterpretation of Einstein's equivalence principle by the Wheeler School [22]. Such a notion is theoretically invalid [13, 18, 22, 24] and recently has been shown as not supported by experiments [58].
7. A difficulty is that mathematicians do not always understand the physical requirements, and physicists do not always understand the related mathematics. For instance, Christodoulou failed in both mathematics and physics [8, 13, 18]. Fields Medalists S. T. Yau (1982) and E. Witten (1990) also follow the invalid assumption of Penrose and Hawking [22] because of their inadequacy in physics. In fact, Yau even overlooked Hawking's logical error at the high school level although it is clearly stated in Hawking's book, "A Brief History of Time".
8. Since the radiation reaction force is very small, the geodesic equation would be an accurate approximation.
9. <u>Mathematically</u>, the non-linear Einstein equation unexpectedly has no bounded dynamic solution [8].
10. <u>Physically,</u> according to the principle of causality, a bounded dynamic solution must exist [12] for a valid equation.
11. According to Veltman [59], one may question whether "spontaneous symmetry breakdown" is really what happens in a non-Abelian gauge theory? However, it is clear that a particular gauge has to be chosen in physics [60].
12. However, theoretical errors are often manifested in so many ways that make a thorough cover up impossible.
13. Errors at the undergraduate level show that D. Christodoulou is inadequate in both mathematics and physics.
14. For instance, Eric J. Weinberg, editor of the Physical Review D, also incorrectly believes that there are dynamic solutions for the Einstein equation [61]. Friedrich W. Hehl, Co-Editor of Annalen der Physik, also incorrectly believes an approximate solution can always be obtained by perturbation [21].
15. To be precise, the case of Poincare' conjecture is completed by Cao & Zhu [62]. Although Grigori Perelman provides a number of sub-conjectures that lead to the completion, there is no evidence that he has done the work. One of his conjectures remains to be proved as valid, in spite of that Perelman had claimed that all have been proved.

References Références Referencias

1. A. Gullstrand, Ark. Mat. Astr. Fys. **16**, No. 8 (1921).
2. A. Gullstrand, Ark. Mat. Astr. Fys. **17**, No. 3 (1922).
3. The 1993 Press Release of the Nobel Prize Committee (The Royal Swedish Academy of Sciences, Stockholm).
4. C. Y. Lo, Essays, **24** (1), 20-27 (2011).
5. C. Y. Lo, Proc. of the Natural Philo. Alliance, 18th Ann. Conf. of the NPA, 6-9 July 2011, p. 354.
6. D. Christodoulou & S. Klainerman, *The Global Nonlinear Stability of the Minkowski Space* (Prin. Univ. Press, 1993); no. 41 of Princeton Mathematical Series.
7. R. M. Wald, *General Relativity* (The Univ. of Chicago Press, Chicago, 1984).
8. C. Y. Lo, Astrophys. J. 455, 421-428 (Dec. 20, 1995); Editor S. Chandrasekhar, a Nobel Laureate, suggests and approves the Appendix: The Gravitational Energy-Stress Tensor for the necessity of modifying Einstein equation.
9. A. Einstein, *The Meaning of Relativity* (1921) (Princeton Univ. Press, 1954).
10. W. Pauli, *Theory of Relativity* (Pergamon Press, London, 1971).
11. C. W. Misner, K. S. Thorne, & J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

12. C. Y. Lo, Astrophys. Space Sci., **306**: 205-215 (2006).
13. C. Y. Lo, Phys. Essays **13** (1), 109-120 (March 2000).
14. Volker Perlick, Zentralbl. f. Math. (827) (1996) 323, entry Nr. 53055.
15. Volker Perlick (republished with an editorial note), Gen. Relat. Grav. 32 (2000).
16. C. Y. Lo, The Principle of Causality and the Cylindrically Symmetric Metrics of Einstein & Rosen, Bulletin of Pure and Applied Sciences, **27D** (2), 149-170 (2008).
17. C. Y. Lo, Misunderstandings Related to Einstein's Principle of Equivalence, and Einstein's Theoretical Errors on Measurements, Phys. Essays **18** (4), 547-560 (December, 2005).
18. C. Y. Lo, Phys. Essays, **13** (4), 527-539 (December 2000).
19. C. Y. Lo, Phys. Essays, **23** (2), 258-267 (2010).
20. R. Schoen and S.-T. Yau, "Proof of the Positive Mass Theorem. II," Commun. Math. Phys. **79**, 231-260 (1981).
21. E. Witten, "A New Proof of the Positive Energy Theorem, "Commun. Math. Phys., **80**, 381-402 (1981).
22. C. Y. Lo, "Could Galileo be Wrong?", Phys. Essays, **24** (4), 477-482 (2011).
23. J. D. Jackson, *Classical Electrodynamics*, (John Wiley & Sons, New York, 1962).
24. C. Y. Lo, Compatibility with Einstein's Notion of Weak Gravity: Einstein's Equivalence Principle and the Absence of Dynamic Solutions for the 1915 Einstein Equation, Phys. Essays **12** (3), 508-526 (Sept. 1999).
25. H. Bondi, F. A. E. Pirani & I. Robinson, Proc. R. Soc. London A **251**, 519-533 (1959).
26. R. Penrose, Rev. Mod. Phys. **37** (1), 215-220 (1965).
27. V. A. Fock, Rev. Mod. Phys. 29, 325 (1957).
28. C. Y. Lo, Einstein's Radiation Formula and Modifications in General Relativity, **The Second William Fairbank Conference**, Hong Kong Polytechnic, Hong Kong Dec. 13-16 (1993).
29. D. Kramer, H. Stephani, E. Herlt, & M. MacCallum, *Exact Solutions of Einstein*'s *Field Equations*, ed. E. Schmutzer (Cambridge Univ. Press, Cambridge, 1980).
30. W. Kinnersley, "Recent Progress in Exact Solutions" in: *General Relativity and Gravitation,* (Proceedings of GR7, Tel-Aviv 1974) ed. G. Shaviv, and J. Rosen (Wiley, New York, London, 1975).
31. A. Peres, Phys. Rev. 118, 1105 (1960).
32. H. W. Ellis and R. R. D. Kemp, remarks in their 1964 lectures.
33. Hogarth, J. E.: *"Particles, Fields, and Rigid Bodies in the Formulation of Relativity Theories*", Ph. D. Thesis 1953, Dept. of Math., Royal Holloway College, University of London (1953), p. 6.
34. Zhou, Pei-Yuan, in *Proc. of the Third Marcel Grossmann Meetings on Gen. Relativ.* ed. Hu Ning, Sci. Press/North Holland. (1983), 1-20.
35. P. Y. Zhou, Proc. of the International Sym. on Experimental Gravitational Physics, Guang Zhou, China (1987).
36. S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 2000).
37. A. Pais, *'Subtle is the Lord …'* (Oxford University press, New York, 1982).
38. C. Y. Lo, Progress in Phys., Vol. 1, 10-13 (Jan., 2006).
39. Comments on Misunderstandings of Relativity, and the Theoretical Interpretation of the Kreuzer Experiment, Astrophys. J. **477,** 700-704 (March 10, 1997).
40. "*Einstein's Miraculous Year*" edited and introduced by John Stachel, Princeton Univ. Press (1998), p. 118.
41. K.S. Thorne, *Black Holes and Time Warps* (Norton, New York, 1994), p. 105.
42. R. Bartnik, J. Math. Phys., 38, 5774 (1997).
43. S. Frittell, & E. T. Newman, Phys. Rev. D, 55, 1971 (1997).
44. E. Malec, J. Math. Phys. 38, 3650 (1997).
45. G. A. Burnett, A. D. Rendall, Class. Quant. 13, 111 (1996).
46. H. Friedrich, Class. Quant. 13, 1451 (1996).
47. A. D. Rendall, Helv. Phys. A, 69, 490 (1996).
48. P. R. Brady, Phys. Rev. D, 51, 4168 (1995).
49. P. Hubner, Class. Quant. 12, 791 (1995).
50. J. Jezierk, Gen. Relat. & Grav., 27, 821 (1995).
51. U. Brauer, A. Rendall & O. Reula, Class. Quant. 11, 2283 (1994).
52. A. D. Rendall, Comm. Math. Phys., 163, 89 (1994).
53. A. Einstein, L. Infeld, & B. Hoffmann, *Annals of Math*. **39** (1): 65-100 (1938).
54. C. Y. Lo, Phys. Essays, **25** (1), 49-56 (2012).
55. C. Y. Lo, Einstein versus the Physical Review on Gravitational Waves and the Principle of Causality, XIII Moscow International Conference on Physical Interpretations of Relativity Theory, Monday 2 July – Thursday 5 July 2007.
56. G. 't Hooft, "**Strange Misconceptions of General Relativity**",http://www.phys.uu.nl/ ~thooft/gravita - ting_misconceptions.html.
57. G.'t Hooft, "A Confrontation with Infinity", Nobel Lecture, December 8, 1999.
58. K-Y. Chung, S-w. Chiow, S. Herrmann, S. Chu, and H. Müller, Phys. Rev. D **80**, 016002 (2009).
59. Martinus J. G. Veltman, "From Weak Interactions to Gravitation", Nobel Lecture, December 8, 1999.
60. C. Y. Lo, Phys. Essays, **23** (3), 491- 499 (Sept. 2010).
61. Eric J. Weinberg, Y. Hu, and M. Turner, Phys. Rev. D49, 3830 (1994).

62. Huai-Dong Cao and Xi-Ping Zhu, "A Complete Proof of the Poincaré and Geometrization Conjectures – application of the Hamilton–Perelman theory of the Ricci flow", vol. 10, no. 2, p. 165–492, *Asian Journal of Mathematics*, June 2006.